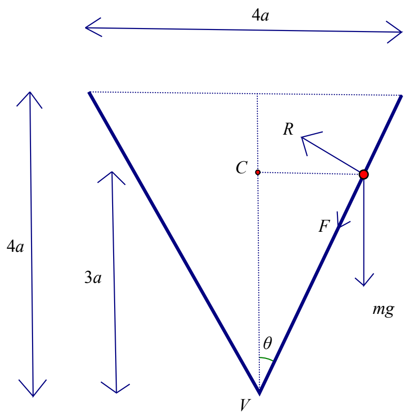
Paper 4F: Further Mechanics 2 Mark Scheme

Question	Scheme	Marks	AOs
1(a)	Total mass = $\int_0^{15} 10\left(1-\frac{x}{25}\right)\mathrm{d}x$	M1	2.1
	$=\left[10x-\frac{x^2}{5}\right]_0^{15}$	A1	1.1b
	$=150-\frac{225}{5}=105\text{ (kg)}$ *	A1*	1.1b
		(3)	
(b)	Taking moments about the base: $\int_0^{15} 10x\left(1-\frac{x}{25}\right)\mathrm{d}x$	M1	3.4
	$=\left[5x^2-\frac{2}{15}x^3\right]_0^{15}\ (=675)$	A1	1.1b
	$\Rightarrow 105d=675$	M1	3.4
	$d=6.43\text{ (m)}\quad 6\frac{3}{7}\text{ (m)}$	A1	1.1b
		(4)	
			(7 marks)

Notes:

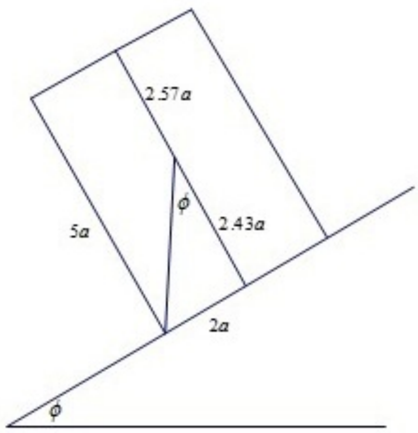
(a)
M1: Use integration (usual rules)
A1: Correct integration
A1*: Use limits and show sufficient working to justify given answer

(b)
M1: Use the model to find the moment about the base (usual rules for integration)
A1: Correct integration
M1: Use the model to complete the moments equation
Require 105 and their 675 used correctly
A1: 6.43 or better

Question	Scheme	Marks	AOs
2			
	Complete overall strategy	M1	3.1b
	Resolve vertically	M1	3.3
	$mg + F\cos\theta = R\sin\theta$	A1	1.1b
	Horizontal equation of motion	M1	3.3
	$mr\omega^2 = R\cos\theta + F\sin\theta$	A1	1.1b
	Use of limiting friction since maximum ω	M1	3.3
	Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$	M1	1.1b
	Maximum $\omega = \sqrt{\frac{3g}{a}}$	A1	1.1b
			(8 marks)

Notes:

M1: Overall strategy to form equation in ω only e.g. consider vertical and horizontal motrion and limiting friction
M1: Needs all 3 terms. Condone sign errors and sin/cos confusion
A1: Correct unsimplified equation
M1: Needs all 3 terms. Condone sign errors and sin/cos confusion
A1: Correct unsimplified equation
M1: Seen or implied
M1: Substitute to achieve equation in a, ω and g only
A1: Or equivalent exact form

Question	Scheme	Marks	AOs
3(a)	<table><tr><td></td><td>mass</td><td>c of m from O</td></tr><tr><td>cylinder</td><td>$4\pi a^2 h$</td><td>$\frac{h}{2}$</td></tr><tr><td>hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr><tr><td>V</td><td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td><td>$d$</td></tr></table>		
	Mass ratios	B1	1.2
	Correct distances	B1	1.2
	Moments about a diameter through O	M1	2.1
	$4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2\left(2h - \frac{1}{3}a\right) \times d$	A1	1.1b
	$d = \dfrac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \dfrac{3\left(8h^2 - a^2\right)}{8\left(6h - a\right)}$ *	A1*	2.2a
		(5)	
(b)			
	$h = 5a \Rightarrow d = 2.573...a$	B1	1.1b
	About to topple so c of m above tipping point	M1	2.2a
	$\Rightarrow \tan\phi = \dfrac{2a}{5a - 2.573a}$	A1ft	1.1b
	$\phi = 39.5°$ **or** 0.689 rads	A1	1.1b
		(4)	
			(9 marks)

Question 3 notes:
(a) **B1:** Correct mass ratios **B1:** Correct distances **M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from *O* **A1:** Correct unsimplified equation **A1*:** Deduce the given answer. Their working must make it clear how they reached their answer
(b) **B1:** Distance of com from base **M1:** Condone tan the wrong way up **A1ft:** Correct unsimplified expression for trig ratio for ϕ following their *d* **A1:** $39.5°$ or 0.689 rads

Question	Scheme	Marks	AOs
4(a)	Equation of motion: $1800 - 2v^2 = 500a$ (when seen)	B1	2.1
	Select form for a: $= 500\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	2.5
	$\int \dfrac{2}{500}\mathrm{d}t = \int \dfrac{1}{900 - v^2}\mathrm{d}v = \dfrac{1}{60}\int \dfrac{1}{30+v} + \dfrac{1}{30-v}\mathrm{d}v$	M1	2.1
	$\dfrac{t}{250} = \dfrac{1}{60}\ln(30+v) - \dfrac{1}{60}\ln(30-v)\ (+C)$	A1	1.1b
	$T = \dfrac{25}{6}\ln\left(\dfrac{30+10}{30-10}\right) = \dfrac{25}{6}\ln 2$ *	M1 A1*	2.1 2.2a
		(6)	
(b)	Equation of motion: $500v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 1800 - 2v^2$	M1	2.5
	$\int \dfrac{500v}{1800 - 2v^2}\mathrm{d}v = \int 1\mathrm{d}x$	M1	2.1
	$-125\ln\left(1800 - 2v^2\right) = x\ (+C)$	A1	1.1b
	Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$	M1	2.1
	$x = 125\ln\dfrac{9}{8}$ (m) *	A1*	2.2a
		(5)	
		(11 marks)	

Notes:

(a)

B1: All three terms & dimensionally correct

M1: Use of correct form for acceleration to give equation in v, t only

M1: Separate variables and integrate

A1: Condone missing C

M1: Use boundary conditions correctly

A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

(b)

M1: Correct form of acceleration in the equation of motion to give equation in v, x only

M1: Separate variables and integrate

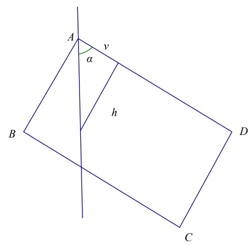
A1: Condone missing C

M1: Extract and use boundary conditions

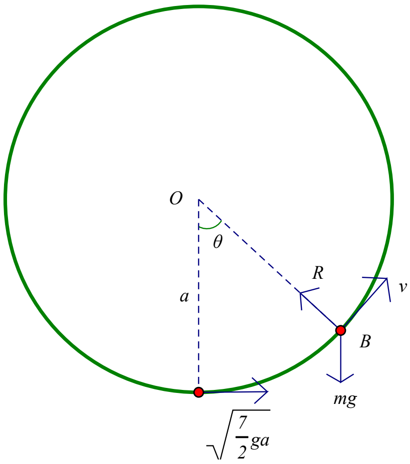
A1*: Show sufficient working to justify given answer and a 'statement' that the required result has been achieved

Question	Scheme	Marks	AOs
5(a)			

	Mass	From AD
Rectangle	$8a^2$	a
Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$
Sign	$a^2\left(8-\frac{\pi}{2}\right)$	h

Question	Scheme	Marks	AOs
	Mass ratios	B1	1.2
	Moments about AD	M1	2.1
	$a^2\left(8-\frac{\pi}{2}\right)h = 8a^2 \times a - \frac{1}{2}\pi a^2 \times \frac{4a}{3\pi}\left(= 8a^3 - \frac{2}{3}a^3 = \frac{22}{3}a^3\right)$	A1	1.1b
	$\Rightarrow h = \frac{22}{3}a \div \left(8-\frac{\pi}{2}\right) = \frac{44a}{3(16-\pi)}$ *	A1*	2.2a
		(4)	
(b)	Moments about A $2aT = \frac{44a}{3(16-\pi)}W$	M1	3.1b
	$T = \frac{hW}{2a} = \frac{22W}{3(16-\pi)}$	A1	1.1b
		(2)	
(c)			
	Take moments about AB to find distance of com from AB	M1	3.1b
	$8a^2 \times 2a - \frac{1}{2}\pi a^2 \times d = \left(8-\frac{1}{2}\pi\right)a^2 \times v$	A1	1.1b
	$v = \frac{32a - \pi d}{16-\pi}$	A1	1.1b
	Correct trig for the given angle	M1	3.1b
	$\tan\alpha = \frac{11}{18} = \frac{h}{v} = \frac{44a}{3(32a-\pi d)}$	A1ft	1.1b
	$(24a = 32a - \pi d,\ \ 8a = \pi d)\quad d = \frac{8a}{\pi}$	A1	1.1b
		(6)	
			(12 marks)

Question 5 notes:	
(a)	
B1:	Correct mass ratios
M1:	Need all three terms, must be dimensionally correct
A1:	Correct unsimplified equation
A1*:	Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved
(b)	
M1:	Could also take moments about B **or** about the c.o.m. and use
A1:	cso
(c)	
M1:	All terms and dimensionally correct
A1:	Correct unsimplified equation
A1:	Or equivalent
M1:	Condone tan the wrong way up
A1:	Equation in a and d; follow through on their v
A1:	cao

Question	Scheme	Marks	AOs
6(a)			
	Conservation of energy	M1	2.1
	$\frac{1}{2}mv^2 + mga(1-\cos\theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$	A1	1.1b
	$v^2 = ga\left(\frac{3}{2} + 2\cos\theta\right)$ *	A1*	2.2a
		(3)	
(b)	Resolve parallel to *OB* and use $\frac{mv^2}{a}$	M1	3.1b
	$R - mg\cos\theta = \frac{mv^2}{a}$	A1	1.1b
	Use R= 0 $\quad g\cos\theta = -\frac{v^2}{a}$	M1	3.1b
	Solve for θ $\Rightarrow$ $g\cos\theta = -g\left(\frac{3}{2} + 2\cos\theta\right)$	M1	1.1b
	$\theta = 120°$	A1	1.1b
		(5)	
(c)	Any appropriate comment e.g. the hoop is unlikely to be smooth	B1	3.5b
		(1)	

Question	Scheme	Marks	AOs
6(d)	At rest $\Rightarrow \quad v = 0$	M1	3.1b
	$\Rightarrow \quad \cos\theta = -\frac{3}{4}$	A1	1.1b
	Acceleration is tangential	M1	3.1b
	Magnitude $\lvert g\cos(\theta - 90)\rvert = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4}g$	A1	1.1b
	At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 = \right) 48.6°$ to the downward vertical	A1	1.1b
		(5)	
		(14 marks)	

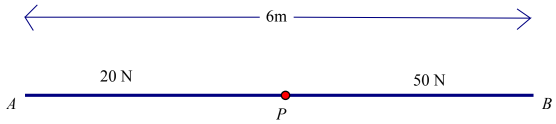
Question 6 notes:

(a)
M1: All terms required. Must be dimensionally correct
A1: Correct unsimplified equation
A1*: Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved

(b)
M1: Resolve parallel to *OB*
A1: Correct equation
M1: Use $R = 0$ seen or implied
M1: Solve for θ
A1: Accept $\frac{2\pi}{3}$

(c)
B1: Any appropriate comment e.g.
- hoop may not be smooth;
- air resistance could affect the motion

(d)
M1: $v = 0$ seen or implied
A1: Correct equation in θ
M1: Correct direction for acceleration
A1: Accept 6.48, 6.5 or exact in g
A1: Accept 0.848 (radians)

Question	Scheme	Marks	AOs
7(a)			
	$T_A = \frac{20e}{2}$, $T_B = \frac{50(2-e)}{2}$ e	M1	3.1a
	In equilibrium $T_A = T_B$, $10e = 25(2-e)$	M1	3.1a
	$(35e = 50)$, $e = \frac{10}{7}$	A1	1.1b
	Equation of motion for P when distance x from equilibrium position towards B:	M1	3.1a
	$3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$	A1 A1	1.1b 1.1b
	$= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$		
	$\Rightarrow 3.5\ddot{x} = -35x$, $\ddot{x} = -10x$ and hence SHM about the equilibrium position	A1	3.2a
		(7)	
(b)	Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$	B1 ft	2.2a
	Use of max speed $= a\omega$	M1	1.1b
	$= \frac{4}{7}\sqrt{10} = 1.81$ (m s^{-1})	A1 ft	1.1b
		(3)	

Question	Scheme	Marks	AOs
7(c)	Nearer to *A* than to *B*: $x < -\frac{3}{7}$	B1	3.1a
	Solve for $\sqrt{10}t$: $\cos\sqrt{10}t = -\frac{3}{4}$, $\sqrt{10}t = 2.418$............	M1	3.1a
	Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418...)$	M1	1.1b
	0.457 (seconds)	A1	1.1b
	Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$		
	Alternative: $x = \frac{4}{7}\sin\sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$ $t_1 = 0.2682,\ t_2 = 0.72527$ $\Rightarrow$ time = 0.457 (seconds)		
		(4)	
		(14 marks)	

Notes:

(a)

M1: Use of $T = \frac{\lambda x}{a}$

M1: Dependent on the preceding M1. Equate their tensions

A1: cao

M1: Condone sign error

A1: Correct unsimplified equation in *e* and *x* A1A1
Equation with one error A1A0

A1: Full working to justify conclusion that it is SHM about the equilibrium position

(b)

B1ft: Seen or implied. Follow their *e*

M1: Correct method for max. speed

A1ft: 1.81 or better. Follow their a, ω

(c)

B1: Seen or implied

M1: Use of $x = a\cos wt$

M1: Correct strategy for the required interval

A1: 0.457 or better